

This round has a nonempty set of hopefully fun puzzles for your team to solve. The round is worth 50 points in total.

## I Twenty Four (5 points)

In each of the problems in this section, you will be given four numbers, and your goal will be to combine them using parentheses and the four basic operations (addition, subtraction, multiplication, and division) to get 24. For example, if the four numbers were 2, 4, 4, 6, one solution would be  $\frac{6(4+4)}{2} = 24$ . Each problem is worth 1 point.

- 1, 2, 3, 7.

**Solution:**  $7 \cdot 3 + 2 + 1 = 24$ .

- 1, 1, 5, 5.

**Solution:**  $1(5 \cdot 5 - 1) = 24$ .

- 4, 4, 10, 10.

**Solution:**  $\frac{10 \cdot 10 - 4}{4} = 24$ .

- 3, 3, 8, 8.

**Solution:**  $\frac{8}{3 - 8/3} = 24$ .

- 1, 5, 5, 5.

**Solution:**  $5(5 - 1/5) = 24$ .

## II Cryptarithmic (3 points)

In the following problems, you will be given some basic arithmetic problems, with the answers worked out. However, all the digits will be replaced by letters, and your job is to find out which letters represent which digits. A letter cannot represent more than one digit, and a digit cannot be represented by more than one letter. For example, if you were given the problem SEND + MORE = MONEY, you would get that  $O = 0, M = 1, Y = 2, E = 5, N = 6, D = 7, R = 8, S = 9$ . When writing your answer, just write the original arithmetic problem, with the letters replaced by numbers. Thus, in this case, you'd write  $9567 + 1085 = 10652$ . Each problem is worth 1 point.

- EAT + THAT = APPLE.

**Solution:**  $819 + 9219 = 10038$ .

- LETS + WAVE = LATER.

**Solution:**  $1567 + 9085 = 10652$ .

- DONALD + GERALD = ROBERT. Hint:  $D = 5$ .

**Solution:**  $526485 + 197485 = 723970$ .

## III Number Puzzles (5 points)

This section will ask you various problems which ask you to find either a particular number or a sequence. Each problem is worth 1 point.

1. Find a 5 digit number such that the result from putting a 1 to the right of the number gives a 6 digit number thrice the number gotten from putting the 1 to the left of the number.

**Solution:** 42857.

2. Arrange the integers from 1 to 16 in a sequence such that the sum of any two consecutive numbers in the sequence is a perfect square.

**Solution:** 16, 9, 7, 2, 14, 11, 5, 4, 12, 13, 3, 6, 10, 15, 1, 8. The reverse sequence is the only other correct solution.

3. Find a 10 digit number, using each digit from 0 to 9 exactly once, such that the number formed by taking the leftmost  $n$  digits is divisible by  $n$ , for all  $n$  between 1 and 10, inclusive. I.e., the number formed from the first digit is divisible by 1, the number formed by the first two digits is divisible by 2, etc.

**Solution:** 3816547290.

4. Find the smallest positive integer which, upon division by any  $n$  between 2 and 10, inclusive, leaves a remainder of  $n - 1$ .

**Solution:** 2519.

5. Find a 4 digit number  $n$  such that the last 4 digits of  $n^2$  are the same as the digits of  $n$ , in the same order.

**Solution:** 9376.

## IV Number Crossword (7 points)

Draw a  $3 \times 3$  grid, and place digits from 1 to 9 such that the resulting 3-digit numbers in the rows (left to right) and columns (top to bottom) satisfy the given properties. You can use each digit more than once or not at all. You will get 7 points for a correct answer and 0 points for an incorrect or incomplete answer.

1st Row: A perfect cube.

2nd Row: A prime number.

3rd Row: A perfect square.

1st Column: A triangular number (of the form  $1 + 2 + 3 + \dots + n$ , for some positive integer  $n$ ).

2nd Column: A perfect fourth power.

3rd Column: A palindrome.

**Solution:** From left to right, top to bottom, the digits are 7, 2, 9, 4, 5, 7, 1, 6, 9. Thus, the rows are, in order, 729, 457, 169; the columns are, in order, 741, 256, 979.

## V Return of the Multiple Choice Test (5 points)

Find the answers to the following questions. You will get 5 points if every answer is correct, and 0 points otherwise.

1. What is the number A's, C's, and E's among the answers?
  - A. 1
  - B. 2
  - C. 3
  - D. 4
  - E. 5
2. Assume A, B, C, D, and E have numerical values of 1, 2, 3, 4, and 5, respectively. What is the sum of the letters of all of the answers?
  - A. 13
  - B. 15
  - C. 17
  - D. 19
  - E. 21
3. What is the first problem in which C is the answer?
  - A. 1
  - B. 2
  - C. 3
  - D. 4
  - E. 5
4. What is the sum of the letters of the answers to the odd-numbered problems, assuming the letters have the same numerical values as given in problem 2?
  - A. 0
  - B. 1
  - C. 2
  - D. 4
  - E. 8
5. What is the answer to this question?
  - A. A
  - B. B
  - C. C
  - D. D
  - E. E

**Solution:** The answers are, in order, C, B, A, E, and D.