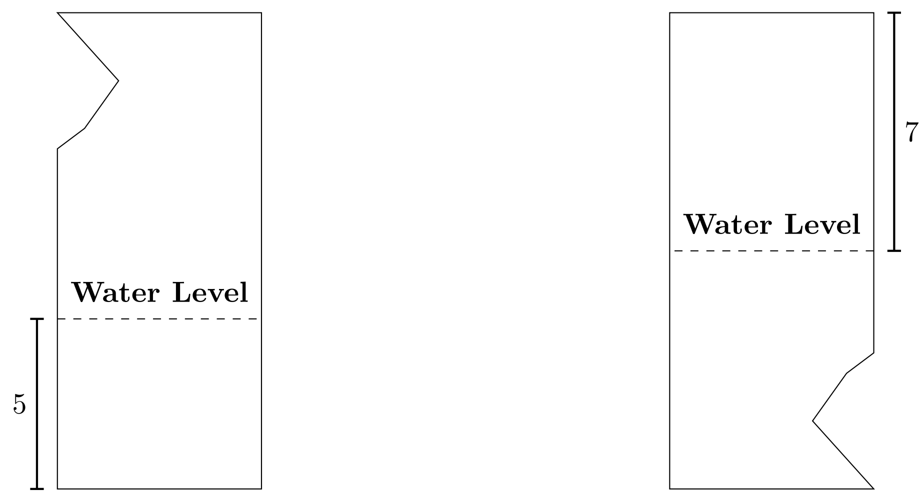
**Time limit:** 30 minutes.
**Instructions:** For this test, you work in teams of five to solve 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written inside the boxes on the answer sheet will be considered for grading.
**No calculators.**

1. Suppose $a_1 \cdot 2 = a_2 \cdot 3 = a_3$ and $a_1 + a_2 + a_3 = 66$. What is $a_3$?

2. Ankit buys a see-through plastic cylindrical water bottle. However, in coming home, he accidentally hits the bottle against a wall and dents the top portion of the bottle (above the 7 cm mark). Ankit now wants to determine the volume of the bottle. The area of the base of the bottle is 20 cm$^2$. He fills the bottle with water up to the 5 cm mark. After flipping the bottle upside down, he notices that the height of the empty space is at the 7 cm mark. Find the total volume (in cm$^3$) of this bottle.




3. If $P$ is a quadratic polynomial with leading coefficient 1 such that $P(1) = 1$, $P(2) = 2$, what is $P(10)$?

4. Let $ABC$ be a triangle with $AB = 1$, $AC = 3$, and $BC = 3$. Let $D$ be a point on $BC$ such that $BD = \frac{1}{3}$. What is the ratio of the area of $BAD$ to the area of $CAD$?

5. A coin is flipped 12 times. What is the probability that the total number of heads equals the total number of tails? Express your answer as a common fraction in lowest terms.

6. Moor pours 3 ounces of ginger ale and 1 ounce of lime juice in cup $A$, 3 ounces of lime juice and 1 ounce of ginger ale in cup $B$, and mixes each cup well. Then he pours 1 ounce of cup $A$ into cup $B$, mixes it well, and pours 1 ounce of cup $B$ into cup $A$. What proportion of cup $A$ is now ginger ale? Express your answer as a common fraction in lowest terms.

7. Determine the maximum possible area of a right triangle with hypotenuse 7. Express your answer as a common fraction in lowest terms.

8. Debbie has six Pusheens: 2 pink ones, 2 gray ones, and 2 blue ones, where Pusheens of the same color are indistinguishable. She sells two Pusheens each to Alice, Bob, and Eve. How many ways are there for her to do so?

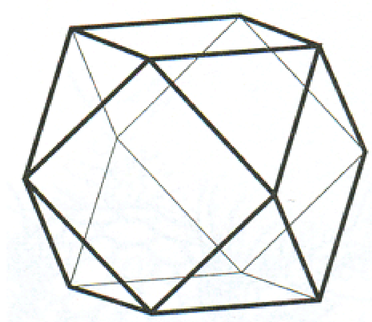
9. How many nonnegative integer pairs $(a, b)$ are there that satisfy $ab = 90 - a - b$?

10. What is the smallest positive integer $a_1 \ldots a_n$ (where $a_1, \ldots, a_n$ are its digits) such that $9 \cdot a_1 \ldots a_n = a_n \ldots a_1$, where $a_1$, $a_n \neq 0$?

11. Justin is growing three types of Japanese vegetables: wasabi root, daikon and matsutake mushrooms. Wasabi root needs 2 square meters of land and 4 gallons of spring water to grow, matsutake mushrooms need 3 square meters of land and 3 gallons of spring water, and daikon need 1 square meter of land and 1 gallon of spring water to grow. Wasabi sell for \$60 per root, matsutake mushrooms sell for \$60 per mushroom, and daikon sell for \$2 per root. If Justin has 500 gallons of spring water and 400 square meters of land, what is the maximum amount of money, in dollars, he can make?

12. A *prim* number is a number that is prime if its last digit is removed. A *rime* number is a number that is prime if its first digit is removed. Determine how many numbers between 100 and 999 inclusive are both prim and rime numbers.

13. Consider a cube. Each corner is the intersection of three edges; slice off each of these corners through the midpoints of the edges, obtaining the shape below. If we start with a $2 \times 2 \times 2$ cube, what is the volume of the resulting solid?



14. If a parallelogram with perimeter 14 and area 12 is inscribed in a circle, what is the radius of the circle?

15. Take a square $ABCD$ of side length 1, and draw $\overline{AC}$. Point $E$ lies on $\overline{BC}$ such that $\overline{AE}$ bisects $\angle BAC$. What is the length of $BE$?

16. How many integer solutions does $f(x) = (x^2 + 1)(x + 2) + (x^2 + 3)(x + 4) = 2017$ have?

17. Alice, Bob, Carol, and Dave stand in a circle. Simultaneously, each player selects another player at random and points at that person, who must then sit down. What is the probability that Alice is the only person who remains standing?

18. Let $x$ be a positive integer with a remainder of 2 when divided by 3, 3 when divided by 4, 4 when divided by 5, and 5 when divided by 6. What is the smallest possible such $x$?

19. A circle is inscribed in an isosceles trapezoid such that all four sides of the trapezoid are tangent to the circle. If the radius of the circle is 1, and the upper base of the trapezoid is 1, what is the area of the trapezoid?

20. Ray is blindfolded and standing 1 step away from an ice cream stand. Every second, he has a $\frac{1}{4}$ probability of walking 1 step towards the ice cream stand, and a $\frac{3}{4}$ probability of walking 1 step away from the ice cream stand. When he is 0 steps away from the ice cream stand, he wins. What is the probability that Ray eventually wins?