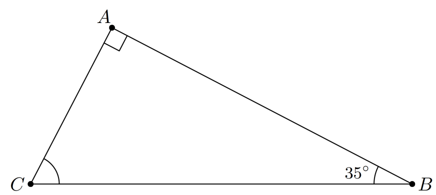
Time limit: 60 minutes.
Instructions: This test contains 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

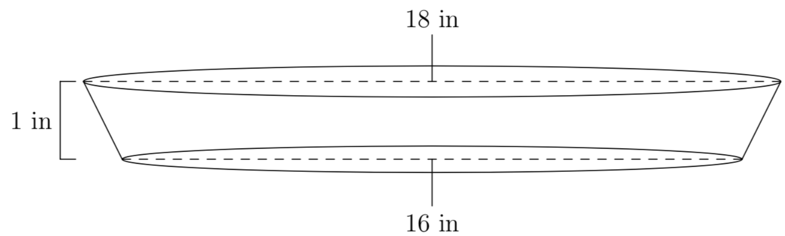
1. If x is a real number that satisfies $\frac{48}{x} = 16$, find the value of x.
2. If ABC is a right triangle with hypotenuse BC such that $\angle ABC = 35^\circ$, what is $\angle BCA$ in degrees?



3. If $a \triangle b = a + b - ab$, find $4 \triangle 9$.
4. Grizzly is 6 feet tall. He measures his shadow to be 4 feet long. At the same time, his friend Panda helps him measure the shadow of a nearby lamp post, and it is 6 feet long. How tall is the lamp post in feet?
5. Jerry is currently twice as old as Tom was 7 years ago. Tom is 6 years younger than Jerry. How many years old is Tom?
6. Out of the $10,000$ possible four-digit passcodes on a phone, how many of them contain only prime digits?
7. It started snowing, which means Moor needs to buy snow shoes for his 6 cows and 7 sky bison. A cow has 4 legs, and a sky bison has 6 legs. If Moor has 36 snow shoes already, how many more shoes does he need to buy? Assume cows and sky bison wear the same type of shoe and each leg gets one shoe.
8. How many integers n with $1 \leq n \leq 100$ have exactly 3 positive divisors?
9. James has three 3 candies and 3 green candies. 3 people come in and each randomly take 2 candies. What is the probability that no one got 2 candies of the same color? Express your answer as a decimal or a fraction in lowest terms.
10. When Box flips a *strange coin*, the coin can land heads, tails, or on the side. It has a $\frac{1}{10}$ probability of landing on the side, and the probability of landing heads equals the probability of landing tails. If Box flips a strange coin 3 times, what is the probability that the number of heads flipped is equal to the number of tails flipped? Express your answer as a decimal or a fraction in lowest terms.
11. James is travelling on a river. His canoe goes 4 miles per hour upstream and 6 miles per hour downstream. He travels 8 miles upstream and then 8 miles downstream (to where he started). What is his average speed, in miles per hour? Express your answer as a decimal or a fraction in lowest terms.

12. Four boxes of cookies and one bag of chips cost exactly 1000 jelly beans. Five bags of chips and one box of cookies cost less than 1000 jelly beans. If both chips and cookies cost a whole number of jelly beans, what is the maximum possible cost of a bag of chips?

13. June is making a pumpkin pie, which takes the shape of a truncated cone, as shown below. The pie tin is 18 inches wide at the top, 16 inches wide at the bottom, and 1 inch high. How many cubic inches of pumpkin filling are needed to fill the pie?



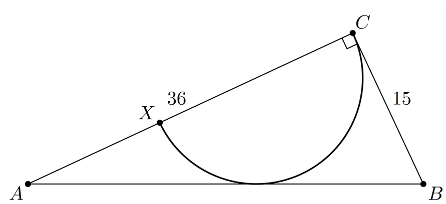
14. For two real numbers a and b, let $a\#b = ab - 2a - 2b + 6$. Find a positive real number x such that $(x\#7)\#x = 82$.

15. Find the sum of all positive integers n such that

$$\frac{n^2 + 20n + 51}{n^2 + 4n + 3}$$

is an integer.

16. Let ABC be a right triangle with hypotenuse AB such that $AC = 36$ and $BC = 15$. A semicircle is inscribed in ABC as shown, such that the diameter XC of the semicircle lies on side AC and that the semicircle is tangent to AB. What is the radius of the semicircle?



17. Let a and b be relatively prime positive integers such that the product ab is equal to the least common multiple of 16500 and 990. If $\frac{16500}{a}$ and $\frac{990}{b}$ are both integers, what is the minimum value of $a + b$?

18. Let x be a positive real number so that $x - \frac{1}{x} = 1$. Compute $x^8 - \frac{1}{x^8}$.

19. Six people sit around a round table. Each person rolls a standard 6-sided die. If no two people sitting next to each other rolled the same number, we will say that the roll is *valid*. How many different rolls are valid?

20. Given that $\frac{1}{31} = 0.\overline{a_1a_2a_3a_4a_5\cdots a_n}$ (that is, $\frac{1}{31}$ can be written as the repeating decimal expansion $0.a_1a_2\cdots a_na_1a_2\cdots a_na_1a_2\cdots$), what is the minimum value of n?