Time limit: 30 minutes.
Instructions: For this test, you work in teams of five to solve 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. What is the sum of the first 12 positive integers?

2. How many positive integers less than or equal to 100 are multiples of both 2 and 5?

3. Alex has a bag with 4 white marbles and 4 black marbles. She takes 2 marbles from the bag without replacement. What is the probability that both marbles she took are black? Express your answer as a decimal or a fraction in lowest terms.

4. How many 5-digit numbers are there where each digit is either 1 or 2?

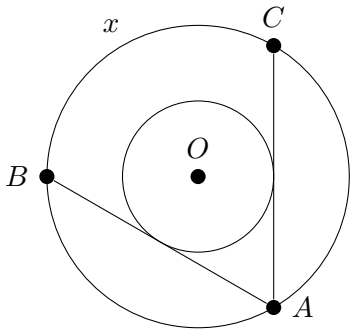
5. An integer a with $1 \leq a \leq 10$ is randomly selected. What is the probability that $\frac{100}{a}$ is an integer? Express your answer as decimal or a fraction in lowest terms.

6. Two distinct non-tangent circles are drawn so that they intersect each other. A third circle, distinct from the previous two, is drawn. Let P be the number of points of intersection between any two circles. How many possible values of P are there?

7. Let x, y, z be nonzero real numbers such that $x + y + z = xyz$. Compute
$$\frac{1+yz}{yz} + \frac{1+xz}{xz} + \frac{1+xy}{xy}.$$

8. How many positive integers less than 10^6 are simultaneously perfect squares, cubes, and fourth powers?

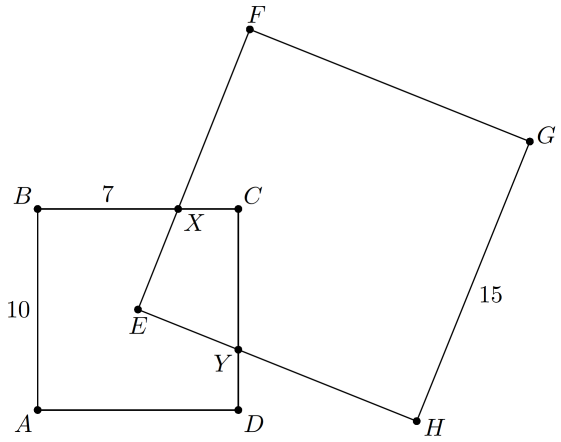
9. Let C_1 and C_2 be two circles centered at point O of radii 1 and 2, respectively. Let A be a point on C_2. We draw the two lines tangent to C_1 that pass through A, and label their other intersections with C_2 as B and C. Let x be the length of minor arc BC, as shown. Compute x.



10. A circle of area π is inscribed in an equilateral triangle. Find the area of the triangle.

11. Julie runs a 2 mile route every morning. She notices that if she jogs the route 2 miles per hour faster than normal, then she will finish the route 5 minutes faster. How fast (in miles per hour) does she normally jog?

12. Let $ABCD$ be a square of side length 10. Let $EFGH$ be a square of side length 15 such that E is the center of $ABCD$, EF intersects BC at X, and EH intersects CD at Y (shown below). If $BX = 7$, what is the area of quadrilateral $EXCY$?

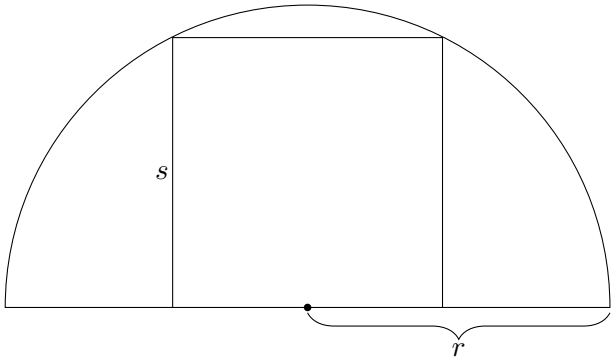


13. How many solutions are there to the system of equations

$$a^2 + b^2 = c^2$$
$$(a+1)^2 + (b+1)^2 = (c+1)^2$$

if a, b, and c are positive integers?

14. A square of side length s is inscribed in a semicircle of radius r as shown. Compute $\frac{s}{r}$.



15. S is a collection of integers n with $1 \leq n \leq 50$ so that each integer in S is composite and relatively prime to every other integer in S. What is the largest possible number of integers in S?

16. Let $ABCD$ be a regular tetrahedron and let W, X, Y, Z denote the centers of faces ABC, BCD, CDA, and DAB, respectively. What is the ratio of the volumes of tetrahedrons $WXYZ$ and $WAYZ$? Express your answer as a decimal or a fraction in lowest terms.

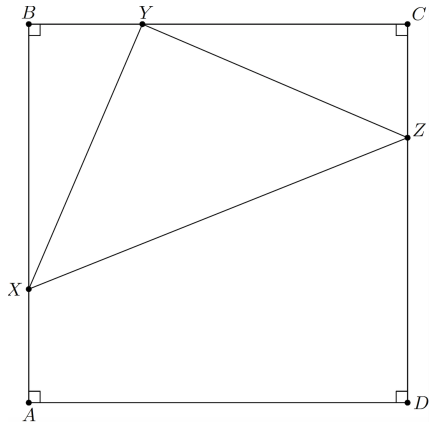
17. Consider a random permutation $\{s_1, s_2, \ldots, s_8\}$ of $\{1, 1, 1, 1, -1, -1, -1, -1\}$. Let S be the largest of the numbers

$$s_1, \quad s_1 + s_2, \quad s_1 + s_2 + s_3, \quad \ldots, \quad s_1 + s_2 + \cdots + s_8.$$

What is the probability that S is exactly 3? Express your answer as a decimal or a fraction in lowest terms.

18. A positive integer is called *almost-kinda-semi-prime* if it has a prime number of positive integer divisors. Given that there are 168 primes less than 1000, how many almost-kinda-semi-prime numbers are there less than 1000?

19. Let $ABCD$ be a unit square and let X, Y, Z be points on sides AB, BC, CD, respectively, such that $AX = BY = CZ$. If the area of triangle XYZ is $\frac{1}{3}$, what is the maximum value of the ratio XB/AX?



20. Positive integers $a \leq b \leq c$ have the property that each of $a + b$, $b + c$, and $c + a$ are prime. If $a + b + c$ has exactly 4 positive divisors, find the fourth smallest possible value of the product $c(c + b)(c + b + a)$.