Time limit: 60 minutes.
Instructions: For this test, you work in teams of five to solve 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. There exist real numbers B, M, and T such that $B + M + T = 23$ and $B - M - T = 20$. Compute $M + T$.

2. Kaity has a rectangular garden that measures 10 yards by 12 yards. Austin's triangular garden has side lengths 6 yards, 8 yards, and 10 yards. Compute the ratio of the area of Kaity's garden to the area of Austin's garden.

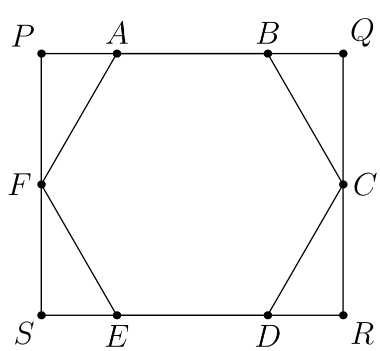
3. Nikhil's mom and brother both have ages under 100 years that are perfect squares. His mom is 33 years older than his brother. Compute the sum of their ages.

4. Madison wants to arrange 3 identical blue books and 2 identical pink books on a shelf so that each book is next to at least one book of the **other** color. In how many ways can Madison arrange the books?

5. Two friends, Anna and Bruno, are biking together at the **same** initial speed from school to the mall, which is 6 miles away. Suddenly, 1 mile in, Anna realizes that she forgot her calculator at school. If she bikes 4 miles per hour faster than her initial speed, she could head back to school and still reach the mall at the same time as Bruno, assuming Bruno continues biking towards the mall at their initial speed. In miles per hour, what is Anna and Bruno's **initial** speed, before Anna has changed her speed? (Assume that the rate at which Anna and Bruno bike is constant.)

6. Let a number be "almost-perfect" if the sum of its digits is 28. Compute the sum of the third smallest and third largest almost-perfect 4-digit positive integers.

7. Regular hexagon $ABCDEF$ is contained in rectangle $PQRS$ such that line $\overline{AB}$ lies on line $\overline{PQ}$, point C lies on line $\overline{QR}$, line $\overline{DE}$ lies on line $\overline{RS}$, and point F lies on line $\overline{SP}$. Given that $PQ = 4$, compute the perimeter of $AQCDSF$.



8. Compute the number of ordered pairs (m, n), where m and n are relatively prime positive integers and $mn = 2520$. (Note that positive integers x and y are relatively prime if they share no common divisors other than 1. For example, this means that 1 is relatively prime to every positive integer.)

9. A geometric sequence with more than two terms has first term x, last term 2023, and common ratio y, where x and y are both positive integers greater than 1. An arithmetic sequence with a finite number of terms has first term x and common difference y. Also, of all arithmetic sequences with first term x, common difference y, and no terms exceeding 2023, this sequence is the longest. What is the last term of the arithmetic sequence?

10. Andrew is playing a game where he must choose three slips, uniformly at random and without replacement, from a jar that has nine slips labeled 1 through 9. He wins if the sum of the three chosen numbers is divisible by 3 and one of the numbers is 1. What is the probability Andrew wins?

11. Circle O is inscribed in square $ABCD$. Let E be the point where O meets line segment $\overline{AB}$. Line segments $\overline{EC}$ and $\overline{ED}$ intersect O at points P and Q, respectively. Compute the ratio of the area of triangle $\triangle EPQ$ to the area of triangle $\triangle ECD$.

12. Define a recursive sequence by $a_1 = \frac{1}{2}$ and $a_2 = 1$, and
$$a_n = \frac{1 + a_{n-1}}{a_{n-2}}$$
for $n \geq 3$. The product
$$a_1 a_2 a_3 \dots a_{2023}$$
can be expressed in the form $a^b \cdot c^d \cdot e^f$, where a, b, c, d, e, and f are positive (not necessarily distinct) integers, and a, c, and e are prime. Compute $a + b + c + d + e + f$.

13. An increasing sequence of 3-digit positive integers satisfies the following properties:
    - Each number is a multiple of 2, 3, or 5.
    - Adjacent numbers differ by only one digit and are relatively prime. (Note that positive integers x and y are relatively prime if they share no common divisors other than 1.)

    What is the maximum possible length of the sequence?

14. Circles O_A and O_B with centers A and B, respectively, have radii 3 and 8, respectively, and are internally tangent to each other at point P. Point C is on circle O_A such that line $\overline{BC}$ is tangent to circle O_A. Extend line $\overline{PC}$ to intersect circle O_B at point $D \neq P$. Compute CD.

15. Compute the product of all real solutions x to the equation $x^2 + 20x - 23 = 2\sqrt{x^2 + 20x + 1}$.

16. Compute the number of divisors of $729,000,000$ that are perfect powers. (A perfect power is an integer that can be written in the form a^b, where a and b are positive integers and $b > 1$.)

17. The arithmetic mean of two positive integers x and y, each less than 100, is 4 more than their geometric mean. Given $x > y$, compute the sum of all possible values for $x + y$. (Note that the geometric mean of x and y is defined to be $\sqrt{xy}$.)

18. Ankit and Richard are playing a game. Ankit repeatedly writes the digits 2, 0, 2, 3, in that order, from left to right on a board until Richard tells him to stop. Richard wins if the resulting number, interpreted as a base-10 integer, is divisible by as many positive integers less than or equal to 12 as possible. For example, if Richard stops Ankit after 7 digits have been written, the number would be 2023202, which is divisible by 1 and 2. Richard wants to win the game as early as possible. Assuming Ankit must write at least one digit, after how many digits should Richard stop Ankit?

19. Eight chairs are set around a circular table. Among these chairs, two are red, two are blue, two are green, and two are yellow. Chairs that are the same color are identical. If rotations and reflections of arrangements of chairs are considered distinct, how many arrangements of chairs satisfy the property that each pair of adjacent chairs are different colors?

20. Four congruent spheres are placed inside a right-circular cone such that they are all tangent to the base and the lateral face of the cone, and each sphere is tangent to exactly two other spheres. If the radius of the cone is 1 and the height of the cone is $2\sqrt{2}$, what is the radius of one of the spheres?