1. Find the value of a satisfying

$$\begin{aligned} a+b&=3\\ b+c&=11\\ c+a&=61 \end{aligned}$$

2. A point P is given on the curve $x^4+y^4=1$. Find the maximum distance from the point P to the origin.

3. Evaluate

$$\lim_{x\to 0}\frac{\sin 2x}{e^{3x}-e^{-3x}}$$

4. Given a complex number z satisfies $\operatorname{Im}(z)=z^2-z$, find all possible values of $|z|$.

5. Suppose that $c_n=(-1)^n(n+1)$. While the sum $\sum_{n=0}^{\infty} c_n$ is divergent, we can still attempt to assign a value to the sum using other methods. The Abel Summation of a sequence, a_n, is $\operatorname{Abel}(a_n)=\lim_{x\to 1^-}\sum_{n=0}^{\infty} a_n x^n$. Find $\operatorname{Abel}(c_n)$.

6. The *minimal polynomial* of a complex number r is the unique polynomial with rational coefficients of minimal degree with leading coefficient 1 that has r as a root. If f is the minimal polynomial of $\cos\frac{\pi}{7}$, what is $f(-1)$?

7. If x,y are positive real numbers satisfying $x^3-xy+1=y^3$, find the minimum possible value of y.

8. Billy is standing at $(1,0)$ in the coordinate plane as he watches his Aunt Sydney go for her morning jog starting at the origin. If Aunt Sydney runs into the First Quadrant at a constant speed of 1 meter per second along the graph of $x=\frac{2}{5}y^2$, find the rate, in radians per second, at which Billy's head is turning clockwise when Aunt Sydney passes through $x=1$.

9. Evaluate the integral

$$\int_0^1 \sqrt{(x-1)^3+1}+x^{2/3}-(1-x)^{3/2}-\sqrt[3]{1-x^2}\,dx$$

10. Let the class of functions f_n be defined such that $f_1(x)=|x^3-x^2|$ and $f_{k+1}(x)=|f_k(x)-x^3|$ for all $k\geq 1$. Denote by S_n the sum of all y-values of $f_n(x)$'s "sharp" points in the First Quadrant. (A "sharp" point is a point for which the derivative is not defined.) Find the ratio of odd to even terms,

$$\lim_{k\to\infty}\frac{S_{2k+1}}{S_{2k}}$$

P1. Prove that for all positive integers m and n,

$$\frac{1}{m}\cdot\binom{2n}{0}-\frac{1}{m+1}\cdot\binom{2n}{1}+\frac{1}{m+2}\cdot\binom{2n}{2}-\cdots+\frac{1}{m+2n}\cdot\binom{2n}{2n}>0$$

P2. If $f(x)=x^n-7x^{n-1}+17x^{n-2}+a_{n-3}x^{n-3}+\cdots+a_0$ is a real-valued function of degree $n>2$ with all real roots, prove that no root has value greater than 4 and at least one root has value less than 0 or greater than 2.