1. Compute the least positive x such that $25x - 6$ is divisible by 1001.

   Answer: 761

   Solution: Just use the Chinese Remainder Theorem, and note that $1001 = 7\times 11\times 13$. Reducing modulo 7, 11, 13, respectively, we have

   $$n \equiv 5 \pmod{7}$$

   $$n \equiv 2 \pmod{11}$$

   $$n \equiv 7 \pmod{13}$$

   Now applying the Chinese remainder theorem, we have

   $$n \equiv \boxed{761} \pmod{1001}$$

2. An integer a is a *quadratic nonresidue* modulo a prime p if there does not exist $x \in \mathbb{Z}$ such that $x^2 \equiv a \pmod{p}$. How many ordered pairs (a, b) modulo 29 exist such that

   $$a + b \equiv 1 \pmod{29}$$

   where both a and b are quadratic nonresidues modulo 29?

   Answer: 7 **Solution: Solution 1:** We shall see that the quadratic nonresidues modulo 29 are

   $$2, 3, 8, 10, 11, 12, 14, 15, 17, 18, 19, 21, 26, 27$$

   We just pick the number of ordered pairs that sum to 30, which is 4, and we have to multiply by 2, and subtract 1 (because $(15, 15)$) to get $\boxed{7}$

   Solution 2: We compute the following sum:

   $$\sum_{j=1}^{28} \left(1 - \left(\frac{j}{29}\right)\right)\left(1 - \left(\frac{1-j}{29}\right)\right) = 28 - 0 - (-1) + (-1) = 28$$

   Now dividing by 4, we get $\boxed{7}$.

3. Let $f : \mathbb{Z}^2 \to \mathbb{C}$ be a function such that $f(x+11, y) = f(x, y+11) = f(x, y)$, and $f(x, y)f(z, w) = f(xz - yw, xw + yz)$. How many possible values can $f(1, 1)$ have?

   Answer: 41

   Solution: Note that the multiplication for the function corresponds to multiplication of complex numbers. This shows that f is multiplicative over the complex numbers which have integer parts. The periodicity conditions suggest that we are working over the integers modulo 11. Note that $f(1, 0)^2 = f(1, 0) \implies f(1, 0) = 0$ or $f(1, 0) = 1$. Also, we can easily compute that the order of $1 + i$ modulo 11 is 40. This means that $f(1, 1)^{40} = f(1, 0)$. If $f(1, 0) = 0$, then $f \equiv 0$. If $f(1, 0) = 1$, then there are 40 values that $(1, 1)$ can be sent to. Hence, the answer is $\boxed{41}$.