Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Define an operation $\diamondsuit$ as $a\diamondsuit b = 12a - 10b$. Compute the value of $((((20\diamondsuit 22)\diamondsuit 22)\diamondsuit 22)\diamondsuit 22)$.

2. The equation
$$4^x - 5 \cdot 2^{x+1} + 16 = 0$$
has two integer solutions for x. Find their sum.

3. Suppose we have four real numbers a, b, c, d such that a is nonzero, a, b, c form a geometric sequence, in that order, and b, c, d form an arithmetic sequence, in that order. Compute the smallest possible value of $\frac{d}{a}$. (A geometric sequence is one where every succeeding term can be written as the previous term multiplied by a constant, and an arithmetic sequence is one where every succeeeding term can be written as the previous term added to a constant.)

4. Find all real x such that
$$\lfloor x \lceil x \rceil \rfloor = 2022.$$
Express your answer in interval notation. (Here, $\lfloor m \rfloor$ is defined as the greatest integer less than or equal to m. For example, $\lfloor 3 \rfloor = 3$ and $\lfloor -4.25 \rfloor = -5$. In addition, $\lceil m \rceil$ is defined as the least integer greater than or equal to m. For example, $\lceil 2 \rceil = 2$ and $\lceil -3.25 \rceil = -3$.)

5. For real numbers B, M, and T, we have $B^2 + M^2 + T^2 = 2022$ and $B + M + T = 72$. Compute the sum of the minimum and maximum possible values of T.

6. The degree-6 polynomial f satisfies $f(7) - f(1) = 1$, $f(8) - f(2) = 16$, $f(9) - f(3) = 81$, $f(10) - f(4) = 256$ and $f(11) - f(5) = 625$. Compute $f(15) - f(-3)$.

7. Let r, s, and t be the distinct roots of $x^3 - 2022x^2 + 2022x + 2022$. Compute
$$\frac{1}{1-r^2} + \frac{1}{1-s^2} + \frac{1}{1-t^2}.$$

8. Given
$$\begin{aligned} x_1 x_2 \cdots x_{2022} &= 1 \\ (x_1+1)(x_2+1)\cdots(x_{2022}+1) &= 2 \\ &\vdots \\ (x_1+2021)(x_2+2021)\cdots(x_{2022}+2021) &= 2^{2021}, \end{aligned}$$
compute
$$(x_1+2022)(x_2+2022)\cdots(x_{2022}+2022).$$

9. We define a sequence $x_1 = \sqrt{3}$, $x_2 = -1$, $x_3 = 2 - \sqrt{3}$, and for all $n \geq 4$
$$(x_n + x_{n-3})(1 - x_{n-1}^2 x_{n-2}^2) = 2x_{n-1}(1 + x_{n-2}^2).$$
Suppose m is the smallest positive integer for which x_m is undefined. Compute m.

10. Let p, q, and r be the roots of the polynomial $f(t) = t^3 - 2022t^2 + 2022t - 337$. Given

$$x = (q-1)\left(\frac{2022-q}{r-1} + \frac{2022-r}{p-1}\right)$$
$$y = (r-1)\left(\frac{2022-r}{p-1} + \frac{2022-p}{q-1}\right)$$
$$z = (p-1)\left(\frac{2022-p}{q-1} + \frac{2022-q}{r-1}\right)$$

compute $xyz - qrx - rpy - pqz$.