Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Compute
$$\frac{\mathrm{d}}{\mathrm{d}x}\sin^2(x) + \frac{\mathrm{d}}{\mathrm{d}x}\cos^2(x).$$

2. Let $f(x) = (x-2)(x-7)^2 + 2x$. Compute the unique real number c not equal to 7 such that $f'(c) = f'(7)$.

3. Compute
$$\int_0^1 e^{x+e^x+e^{e^x}}\,\mathrm{d}x.$$

4. Let $f(x)$ be a degree-4 polynomial such that $f(x)$ and $f'(x)$ both have 20 and 22 as roots. Given that $f(21) = 21$, compute $f(23)$.

5. Compute
$$\sum_{n=0}^{\infty}\left(\sqrt{n^2+3n+2} - \sqrt{n^2+n} - 1\right).$$

6. Compute
$$\int_0^{\pi/3} \sec(x)\sqrt{\tan(x)\sqrt{\tan(x)\sqrt{\tan(x)\sin(x)}}}\,\mathrm{d}x.$$

7. Compute
$$\lim_{x\to 0}\left(1 + \int_0^x \frac{\cos(t)-1}{t^2}\,\mathrm{d}t\right)^{1/x}.$$

8. At the Berkeley Mart for Technology, every item has a real-number cost independently and uniformly distributed from 0 to 2022. Sumith buys different items at the store until the total amount he spends strictly exceeds 1. Compute the expected value of the number of items Sumith buys.

9. Compute
$$\int_0^{\frac{\pi}{2}} \cot(x)\ln(\cos(x))\,\mathrm{d}x,$$
where ln denotes the natural logarithm.

10. A unit cube is rotated around an axis containing its longest diagonal. Compute the volume swept out by the rotating cube.