Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. For lunch, Lamy, Botan, Nene, and Polka each choose one of three options: a hot dog, a slice of pizza, or a hamburger. Lamy and Botan choose different items, and Nene and Polka choose the same item. In how many ways could they choose their items?

2. Compute the number of positive integer divisors of 100000 which do not contain the digit 0.

3. Katie and Allie are playing a game. Katie rolls two fair six-sided dice and Allie flips two fair two-sided coins. Katie's score is equal to the sum of the numbers on the top of the dice. Allie's score is the product of the values of two coins, where heads is worth 4 and tails is worth 2. What is the probability Katie's score is strictly greater than Allie's?

4. Richard and Shreyas are arm wrestling against each other. They will play 10 rounds, and in each round, there is exactly one winner. If the same person wins in consecutive rounds, these rounds are considered part of the same "streak". How many possible outcomes are there in which there are strictly more than 3 streaks? For example, if we denote Richard winning by R and Shreyas winning by S, $SSRSSRRRRR$ is one such outcome, with 4 streaks.

5. Given a positive integer n, let $s(n)$ denote the sum of the digits of n. Compute the largest positive integer n such that $n = s(n)^2 + 2s(n) - 2$.

6. Bayus has eight slips of paper, which are labeled 1, 2, 4, 8, 16, 32, 64, and 128. Uniformly at random, he draws three slips with replacement; suppose the three slips he draws are labeled a, b, and c. What is the probability that Bayus can form a quadratic polynomial with coefficients a, b, and c, in some order, with 2 distinct real roots?

7. Luke the frog has a standard deck of 52 cards shuffled uniformly at random placed face down on a table. The deck contains four aces and four kings (no card is both an ace and a king). He now begins to flip over the cards one by one, leaving a card face up once he has flipped it over. He continues until the set of cards he has flipped over contains at least one ace and at least one king, at which point he stops. What is the expected value of the number of cards he flips over?

8. Define the two sequences $a_0, a_1, a_2, \ldots$ and $b_0, b_1, b_2, \ldots$ by $a_0 = 3$ and $b_0 = 1$ with the recurrence relations $a_{n+1} = 3a_n + b_n$ and $b_{n+1} = 3b_n - a_n$ for all nonnegative integers n. Let r and s be the remainders when a_{32} and b_{32} are divided by 31, respectively. Compute $100r + s$.

9. Lysithea and Felix each have a take-out box, and they want to select among 42 different types of sweets to put in their boxes. They each select an even number of sweets (possibly 0) to put in their box. In each box, there is at most one sweet of any type, although the boxes may have sweets of the same type in common. The total number of sweets they take out is 42. Let N be the number of ways can they select sweets to take out. Compute the remainder when N is divided by $42^2 - 1$.

10. Compute the number of integer ordered pairs (a, b) such that $10!$ is a multiple of $a^2 + b^2$.